On some property of the stochastic derivative operator

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Starting from the 70th of the past century, many attempts were made to weak the requirement for the integrand to be adapted for the integrand of the Ito's stochastic integral as well as in the theory of "the extension of filtration". Skorokhod (1975) suggested absolutely different method, symmetric with respect to the time inversion and did not require for the integrand to be independent of the future Wiener process. Towards this end, he required for the integrand to be smooth in a certain sense, i.e., its stochastic differentiability. This idea was later on developed in the works of Protter, Malliavin (1979), Gaveau-Trauber (1982), Nualart, Zakai (1986), Pardoux (1982), etc. It turned out (as it was shown by Gaveau and Trauber in 1982) that the operator of Skorokhod stochastic integration coincides with the conjugate operator of stochastic differentiation in the sense of Malliavin. Ocone (1984) have noticed that the integrand of the so called Clark representation (or, in general, the martingale representation) may be expressed by means of the stochastic derivative operator. At last, Harison and Pliska (1981) have established that the martingale representation theorems (along with the Girsanov's measure change theorem) play an important role in the modern financial mathematics.

We investigate some new properties of stochastic derivative operator for the class of Wiener functionals for the purpose of the subsequent application in financial mathematics. In particular, we offer a method of finding of kernels of multiple stochastic integrals in chaotic decomposition of “average” stochastic process, when the corresponding kernels of initial square integrable process are known; we proof that if the square integrable random process is not stochastic differentiable, then the "average" process also is not stochastic differentiable; the rule of stochastic differentiation of a composite function is obtained and property of a commutativity of the stochastic derivative operator and the Sobolev’s average operator is proved.

Research partially supported by Shota Rustaveli National Scientific Grants No FR/308/5-104/12, FR/69/5-104/12.