Continuity and differentiability of solution with respect to initial element for some classes of functional differential equations

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For every initial element $\mu = (\varphi(t), \tau(t), f)$, which is the collection of initial function, delay function and the right-hand side of equation, we assign the functional differential equation

$$\dot{x}(t) = f(t, x(t), x(\tau(t))), t \in [t_0, t_1] \quad (1)$$

with the continuous initial condition

$$x(t) = \varphi(t), \ t \in [\tau(t_0), t_0], \quad (2)$$

where t_0 and t_1 are fixed initial and final moments. By $x(t;\mu)$ we denote a solution of the problem (1)-(2). Consider the mapping

$$\mu \to x(t;\mu) \quad (3)$$

In the paper continuity of mapping (3) is proved and its differential is calculated. These assertions are used under investigation of optimal control problems [1] and for sensitivity analysis of mathematical models in the immunology [2]. Moreover, analogous results are obtained for the quasi-linear neutral controlled functional differential equation

$$\dot{x}(t) = A(t)\dot{x}(\sigma(t)) + f(t, x(t), x(\tau(t)), u(t)), t \in [t_0, t_1],$$

where $u(t)$ is a control function. In this case initial element is the collection of initial function, delay and control functions.

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References

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