ON THE $Q_{1,N}$-DEGREES OF r-MAXIMAL SETS

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Tennenbaum (see, [4, p.159]) defined the notion of Q-reducibility on stes of natural numbers as follows: a set A is Q-*redusible* to a set B (in symbols: $A \leq_Q B$) if there exists a computable function f such that for every $x \epsilon \omega$ (where ω denotes the set of natural numbers),

$$x \epsilon A \Leftrightarrow W_{f(x)} \subseteq B.$$

We say in this case that $A \leq_Q B$ via f. If $A \leq_Q B$ via a computable function f such that for all x, y,

$x \neq y \Rightarrow W_{f(x)} \cap W_{f(y)} = \emptyset$ and $\bigcup_{x \epsilon \omega} W_{f(x)}$ is computable, then we say that A is $Q_{1,N}$-*reducible* to B, and denoted $A \leq_{Q_{1,N}} B$. The notion of $Q_{1,N}$ reducibility was introduced by Bulitko in [1].

A c.e. set M is r-*maximal* if $\overline{M}$ is infinite and for every computable R, either $R \cap \overline{M}$ or $\overline{R} \cap \overline{M}$ is finite.

If A is any noncomputable c.e. set, *a nontrivial splitting* of A is a pair of disjoint noncomputable c.e. sets A_0, A_1 such that $A = A_0 \cup A_1$.

A set A is *hemi r-maximal* (see, [2]) if there are a r-maximal set M and a nontrivial splitting M_0, M_1 of M such that $A = M_0$.

Our notation and terminology are standard and can be found in [4].

Theorem 1. Let M be an r-maximal set, A be an arbitrary set and $M \equiv_{Q_{1,N}} A$. Then $M \leq_m A$.

Given c.e. sets $A \subseteq B$, A is major subset of B (written $A \subset_m B$) (see, [3]) if B–A is infinite and for every c.e. set W, $\overline{B} \subseteq^* W \Rightarrow \overline{A} \subseteq^* W$.

Theorem 2. Let M be an r-maximal set, A be a major subset of M, B be an arbitrary set and M–A $\equiv_{Q_{1,N}} B$. Then $M \leq_m B$.

Theorem 3. If C, D are hemi r-maximal sets then

$$C \equiv_{Q_{1,N}} D \Leftrightarrow C \equiv_1 D.$$

[1] V.K.Bulitko, On ways of characterizing complete sets, Math. USSR, Izv. vol 38 (1992), no.2.

[2] R.G.Downey and M.Stob, Automorphisms of the lattice of recursively enumerable sets: Orbits. Advances in Mathematics, vol.92 (1992).

[3] A.H.Lachlan, On the lattice of recursively enumerable sets, Trans. Amer. Math. Soc.,130, 1 (1968).

[4] H.Rogers, Theory of recursive functions and effective computability. McGraw-Hill Book Co., New York, 1967.