6D Standing Wave Braneworld with Ghost Scalar Fields

Otari Sakhelashvili

Javakhishvili Tbilisi State University
3 Chavchavadze Avenue, Tbilisi 0179, Georgia
E-mail: otosaxel@gmail.com

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Abstract

The 6D braneworld with the phantom-like bulk scalar field is considered. We demonstrate pure gravitational localization of scalar field zero modes on the brane.

Keywords: Brane; Standing waves; Zero modes

1 Introduction

Braneworld models involving large extra dimensions [1, 2] have been very useful in addressing several open questions in high energy physics (see [3] for reviews). Most of the braneworlds are realized as time-independent field configurations. However, there have appeared several braneworld models that assumed time-dependent metrics and fields [4, 5, 7].

In this paper we study the non-stationary 6D braneworld generated by standing gravitational waves coupled to a phantom-like bulk scalar field. The model is generalization of 5D standing waves scenario of [5]. The metric we use is special case of the general solution found in [6] and slightly differs from those of the 6D models [7].

2 The solution of Einstein equations

We consider 6D space-time, having the signature $(+, -, -, -, -, -)$, with a non-self interacting phantom-like scalar field coupled to gravity. The action of the model is:

$$S = \int d^6 x \sqrt{g} \left(\frac{M^4}{2} R - \frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi \right) , \qquad (1)$$

where M is the fundamental scale, which relates to the 6D Newton constant, $G = 1/(8\pi M^4)$. Capital Latin indexes numerate the coordinates of 6D space-time and we use the units where $c = \hbar = 1$. Variation of the action (1) with respect to g_{AB} leads to the 6D Einstein equations:

$$R_{AB} - \frac{1}{2} g_{AB} R = \frac{1}{M^4} T_{AB} , \qquad (2)$$

where

$$T_{AB} = -\partial_A\phi\partial_B\phi + \frac{1}{2}g_{AB}\partial^C\phi\partial_C\phi \ . \tag{3}$$

is the energy momentum tensor of the scalar field. Using (3) the Einstein equations (2) can be rewritten in the form:

$$R_{AB} = -\frac{1}{M^4}\partial_A\phi\partial_B\phi \ . \tag{4}$$

To solve the equations (4) we take the metric *ansatz*:

$$ds^2 = e^S dt^2 - e^u \left(dx^2 + dy^2 + dz^2\right) - dr^2 - e^{-3u}d\theta^2 \ , \tag{5}$$

where $S = S(r)$, $u = u(t, |r|)$ are some functions, and suppose that phantom-like scalar field depends only on the time and on the modulus of one extra dimension coordinate r. By the metric (5) we want to describe geometry of the brane placed at the origin of the large extra dimension, r, when x, y and z denote coordinates along the brane. The sixth coordinate θ can be assumed to be compact, curled up to the unobservable sizes for the present energies.

Advantage of our 6D *ansatz* (5) is that it looks symmetric in the brane coordinates x, y and z, while in 5D model [5] the brane 3-plain is non-symmetrically warped. Also (5) is simpler than the metric of the models [7], since it contains only one singular point, $r = 0$, where the brane is placed to smooth the singularity.

The system of Einstein equations (4) for the *ansatz* (5) takes the form:

$$\begin{aligned}
3\dot{u}^2 - \frac{1}{2}S''e^S - \frac{1}{4}S'^2 e^S &= \frac{1}{M^4}\dot{\phi}^2 \ , \\
3\dot{u}u' &= \frac{1}{M^4}\dot{\phi}\phi' \ , \\
\frac{1}{2}S'' + \frac{1}{4}S'^2 + 3u'^2 &= \frac{1}{M^4}\phi'^2 \ , \\
-2\ddot{u} + S'e^S u' + 2u''e^S &= 0 \ ,
\end{aligned} \tag{6}$$

where overdots and primes mean derivatives with respect to t and $|r|$, respectively. If we assume that the metric field, u, is proportional to the bulk phantom field, ϕ, and separate the variables:

$$u(t, |r|) = \frac{1}{\sqrt{3M^4}} \ \phi(t, |r|) = \sin(\omega t)f(|r|) \ , \tag{7}$$

the system (6) reduces to:

$$\begin{aligned}
S'' + \frac{1}{2}S'^2 &= 0 \ , \\
2\omega^2 f + S'e^S f' + 2f''e^S &= 0 \ .
\end{aligned} \tag{8}$$

Solution of the first equation of (8) with the boundary condition:

$$S(0) = 0 \ , \tag{9}$$

(leading to the Minkowski metric at $r = 0$) is;

$$S = \ln\left(1 + \frac{|r|}{a}\right)^2 \ , \tag{10}$$

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where a is some constant. Using (10) the second equation of (8) takes the form:

$$f'' + \frac{1}{a + |r|}f' + \frac{a^2\omega^2}{(a + |r|)^2}f = 0 \ . \tag{11}$$

Solution to this equation leading to the Minkowski metric at $r = 0$ is:

$$f(|r|) = C \sin\left(a\omega \ln\left[1 + \frac{|r|}{a}\right]\right) \ , \tag{12}$$

where C is a constant.

Finally the metric function (7) obtains the form:

$$u(t, |r|) = C \sin(\omega t) \sin\left(a\omega \ln\left[1 + \frac{|r|}{a}\right]\right) \ , \tag{13}$$

and the solution of Einstein equations for our *ansatz* (5) is:

$$\begin{aligned} ds^2 &= (1 + |r|/a)^2 \, dt^2 - e^{C\sin(\omega t)\sin(a\omega \ln[1+|r|/a])}\left(dx^2 + dy^2 + dz^2\right) - \\ &- dr^2 - e^{-3C\sin(\omega t)\sin(a\omega \ln[1+|r|/a])}d\theta^2 \ . \end{aligned} \tag{14}$$

3 Time averages

When the frequency of standing gravitational waves, ω, is much larger than frequencies associated with the energies of the particles on the brane,

$$\omega \gg E \ , \tag{15}$$

we can perform time averaging of oscillating exponents in the equations of matter fields. Using the known formula for averages:

$$\left\langle e^{D\sin(x)}\right\rangle = I_0(D) \ , \tag{16}$$

where I_0 is modified Bessel function of zero order, we can find the time average of our *ansatz* (5):

$$\begin{aligned} \left\langle ds^2\right\rangle &= (1 + |r|/a)^2 dt^2 - I_0(C\sin\left(a\omega \ln[1 + |r|/a]\right))\left(dx^2 + dy^2 + dz^2\right) - \\ &- dr^2 - I_0(-3C\sin\left(a\omega \ln[1 + |r|/a]\right))d\theta^2 \end{aligned} \tag{17}$$

4 Localization of scalar fields

Let us consider the localization problem for the massless scalar field, Φ, defined by the 6D action:

$$S_\Phi = \frac{1}{2}\int d^6x \sqrt{g}g^{MN}\partial_M\Phi\partial_N\Phi \ . \tag{18}$$

The corresponding Klein-Gordon equation is:

$$\frac{1}{\sqrt{g}}\, \partial_M\left(\sqrt{g}g^{MN}\partial_N\Phi\right) = 0 \ . \tag{19}$$

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We look for the solution of this equation in the form:

$$\Phi(x^A) = \psi(t, x, y, z) \sum_l \nu_l(r) e^{il\theta} \tag{20}$$

Let us consider the S-wave solution ($l = 0$) and nothing depends on the extra dimension angle θ. After inserting of (20) into the action (18) and performing of time averaging we find:

$$S_\Phi = \frac{1}{2} \int d^6 x \left\{ \frac{\nu^2 (\partial_t \psi)^2}{1 + r/a} - \left(1 + \frac{r}{a}\right) \nu'^2 \psi^2 - \left(1 + \frac{r}{a}\right) \nu^2 \langle e^u \rangle \left[(\partial_x \psi)^2 + (\partial_y \psi)^2 + (\partial_z \psi)^2\right] \right\}. \tag{21}$$

In general, to have a field localized on a brane 'coupling' constants appearing after integration of the Lagrangian over the extra coordinates must be non-vanishing and finite. So normalizable zero modes of our scalar field, Φ, on the brane will be exists if the action (21) is integrable over r, i.e. the functions $(1 + r/a)\nu'^2$, $(1 + r/a)\langle e^u \rangle \nu^2$ and $\frac{1}{1+r/a}\nu^2$ are integrable.

On the background (14) the differential operator in the Klein-Gordon equation (19), after time averaging, gets the form:

$$\widehat{L} = \frac{1}{(1 + r/a)}\frac{\partial^2}{\partial t^2} - \left(1 + \frac{r}{a}\right)\langle e^{-u} \rangle \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) - \frac{\partial}{\partial r}\left(1 + \frac{r}{a}\right)\frac{\partial}{\partial r} - \langle e^{3u} \rangle \frac{\partial^2}{\partial \theta^2} . \tag{22}$$

Applying this operator for the S-wave part of the scalar field wavefunction (20) we find:

$$\left[\left(1 + \frac{r}{a}\right)\nu(r)'\right]' + \left[\frac{1}{(1 + r/a)}E^2 - \left(1 + \frac{r}{a}\right)\langle e^u \rangle \left(p_x^2 + p_y^2 + p_z^2\right)\right]\nu(r) = 0 . \tag{23}$$

Transformation of the variable,

$$\nu(r) = \frac{\phi}{\sqrt{1 + r/a}} , \tag{24}$$

brings (23) to the Schrodinger-like form:

$$\phi'' - U(r)\phi = 0 . \tag{25}$$

Here the potential function is:

$$U(r) = \langle e^{-u} \rangle \left(p_x^2 + p_y^2 + p_z^2\right) - \frac{a^2 E^2}{(a + r)^2} - \frac{1}{4(a + r)^2} , \tag{26}$$

Figure 1 displace the shape of $U(r)$ for the positive r-s.

If we would assume:

$$\langle e^{-u} \rangle \left(p_x^2 + p_y^2 + p_z^2\right) \approx P^2 . \tag{27}$$

Close to the brane $r \ll a$ and the equation (26) takes the form:

$$\phi'' - \left(P^2 - E^2 - \frac{1}{4a^2}\right)\phi = 0 . \tag{28}$$

Close to the origin $r \to 0$, i.e. on the brane, this equation gives the standard dispersion relation,

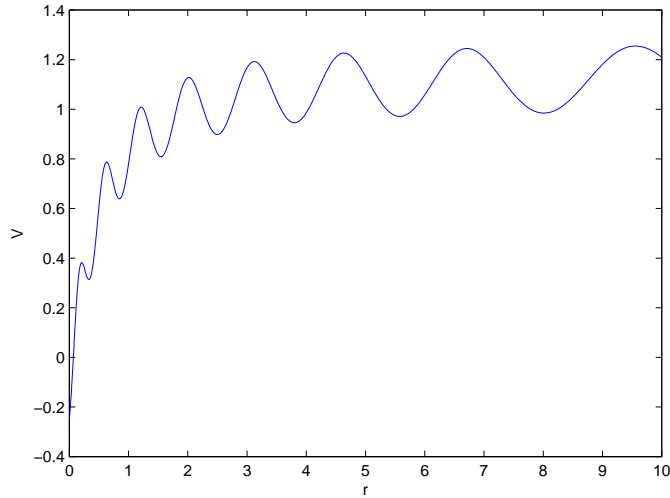$$E^2 = p_x^2 + p_y^2 + p_z^2 , \tag{29}$$

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Figure 1: Shape of the effective bulk potential $U(r)$ for scalar zero mode.

for zero mode particle. It is easy to find that the equation (28) has the plane wave solution:

$$\phi|_{r \to 0} = C_1 e^{ir/2a} + C_2 e^{-ir/2a} \ , \tag{30}$$

where C_1 and C_2 are constants.

Fare from the brane, $r \to \infty$, the equation (26) reduces to:

$$\phi'' - P^2 \phi = 0, \tag{31}$$

So at the infinity ϕ behaves as

$$\phi|_{r \to \infty} = B_1 e^{r/P} + B_2 e^{-r/P} \ , \tag{32}$$

where B are another integration constants. So if we assume that $B_1 = 0$ the function ϕ will be sharply decreasing function from the location of the brane, i.e. scalar field Φ will be localized on the brane.

We have demonstrate pure gravitational localization of scalar zero mode particles on the brane using approximate solutions (30) and (32). We can obtain also numerical solutions to the equations (23). We use Runge-Couta 4-5 Scheme, with 10^{-11} error tolerance.

Figure 2 displays the behaviour of the function $\nu(r)$ and its first derivative close to the brane.

To proof localization of Φ we need to show that all integrals over r in its action (21) is finite. On Figure 3 we display all r-depended factors in (21) which are multiplied by r. We see that they all are decreasing functions, i.e. (21) is integrable over r and scalar field zero modes are localized on the brane.
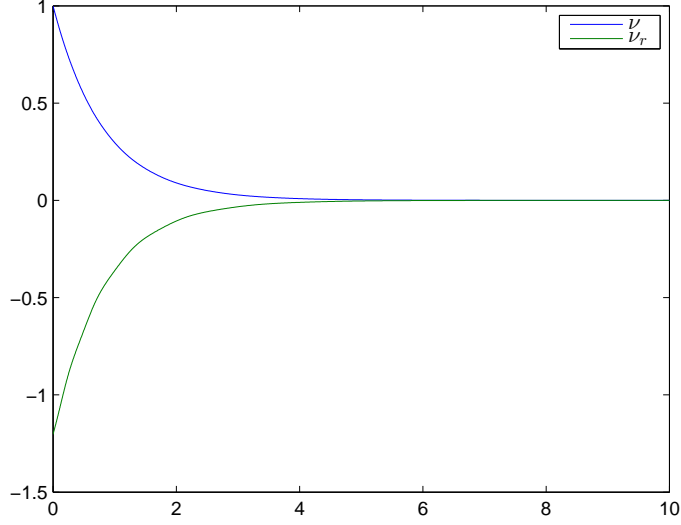
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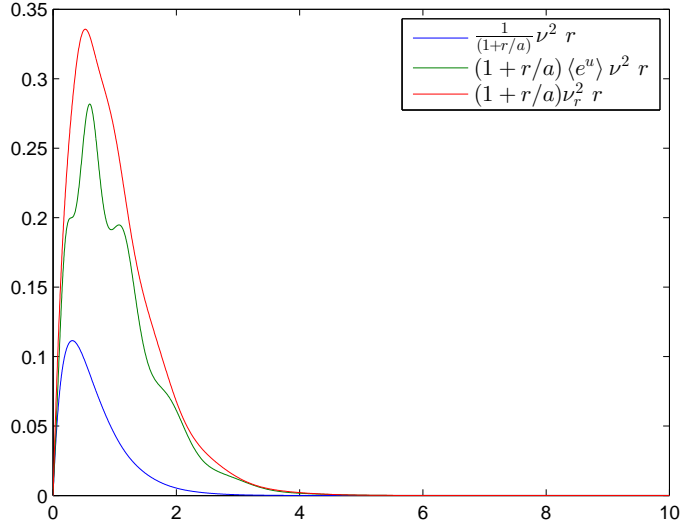Figure 2: Numerical solutions for ν and ν'.

Figure 3: r-depended factors in (21) multiplied by r

5 Conclusions

In this letter we have demonstrated the pure gravitational localization of scala field zero modes within the 6D model of the standing wave braneworld [5]. The model represents a single (1+3)-brane in six dimensional space-time with one large (infinite) and one small (compact) space-like extra dimensions. The trapping is provided by the rapid oscillations of gravitational; and ghost-

like scalar fields in the bulk. The main differences of our model from 5D case is that the plane of the brane is warped symmetrically.

Acknowledgments

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