To Secular Equation

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As is known the secular equations is

$$\det|A-\lambda E| = \lambda^n - p_1\lambda^{n-1} - p_2\lambda^{n-2} - \ldots - p_{n-1}\lambda - p_n = 0, \quad (1)$$

where A is square matrix, E -- unit one, roots of (1)- named as eigenvalues.

Let the given matrix A presented as $A_1 + r_i(\varepsilon)A_2$, where A_i are the matrices with integer elements, $|r_i(\varepsilon)| < 1, \alpha = r_i(\varepsilon)$ is ultra-spherical classical polynomial of degree i. We now applied Leverier-Faddeev method with respect to *mathrices* A_i expressed coefficients as matrix polynomials and parameter $r_i(\varepsilon)$.This process guaranteed high order of accuracy of calculation as so the matrix polynomials are calculated exactly (in bounds of memory of recent PC) and for Special functions with R.Chikashua are created and realizing algorithms and package of programs.

The important step for successful functioning of above schemes is the reliable calculation of classical orthogonal polynomials of degree of order 10^6 (million) with 1200 (one thousand two hundred) decimal signs. The ending process contains to calculate the eigenvalues of (1) by high order method corresponding to above schemes. There is evident that the scheme of defining coefficients may be develop if we use well known method of Dandelen-Greffe-Lobachevski(DGL). By this method which contains p stages, are defining roots of (1) in degree of $m = 2^p$ .The operations of taking the roots are rough with respect to round of errors .In this connection there is sufficient better to use algorithms of Lehmer (D.H.Lehmer, Acta Matematika, v.95, N3-4, 1956) representing modification of method of DGL .By these algorithms schemes of same are true and give eigenvalues of (1) immediately.